# Invariant submanifolds of LCS-manifold admitting semi-symmetric metric connection

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**Abstract: In this paper, we study invariant submanifolds of $(LCS)$-manifold admitting semi-symmetric metric connection. We prove that the submanifolds also carry the semi-symmetric metric connection. Further, we also consider recurrent, 2-recurrent and generalized 2-recurrent submanifolds of $(LCS)$-manifold admitting semi-symmetric metric connection and we investigate the conditions for all the above mentioned submanifolds to be totally geodesic.**

**Key Words:** ***Invariant submanifold, (LCS)-manifold, recurrent, 2-recurrent, generalized 2-recurrent and totally geodesic.***

## I. INTRODUCTION

Let $(M,g)$ be an n-dimensional Riemannian manifold with metric $g$ and Let $\nabla$ be the Levi-civita connection on $M$. A linear connection $\bar{\nabla}$ on $(M,g)$ is said to be Semi-symmetric [1] if the torsion tensor $T$ of connection $\bar{\nabla}$ satisfies

$$T(X,Y) = W(X)Y - W(Y)X$$

where $W$ is a 1-form on $M$. Further let $\rho$ be the vector field associated with it, that is $W(X) = g(X,\rho)$ for any differentiable vector field $X$ on $M$. A Semi-symmetric connection $\bar{\nabla}$ is called Semi-symmetric metric connection [2] if it satisfies $\bar{\nabla}g = 0$. In 1924, Friedman and Schouten [1] introduced the idea of semi-symmetric linear connection on a differentiable manifold. Further in 1932, The idea of metric connection with tortion on a Riemannian manifold was introduced by H.A. Hayden [2]. Later, Yano [3] synthesized the notion of semi-symmetric connection and a metric connection with torsion on a Riemannian manifold in 1970. After the properties of semi-symmetric metric connection have studied by many authors like K.S. Amur and S.S. Pujar [4],C.S. Bagewadi, D.G. Prakasha and Venkatesha [5,6], A. Sharfuddin and S.I. Hussain[7], U.C. De and G. Pathak [8] etc. Recently, in 2003 A.A. Shaikh [9] intro- duced and studied Lorentzian concircular structure manifold (briefly (LCS)-manifold) which generalizes the notion of LP-sasakian manifolds, introduced by Matsumoto [10] in 1989. Then Shiakh and Baishya [11,12] have been studied on the applications of $(LCS)_n$- manifolds to the general theory of relativity and cosmology. It is to be noted that $(LCS)_n$ manifold remains invariant under a D-homothetic transformation [13]. On the other hand in 2008, A.A. Shaikh, T.Basu and S. Eyasmin [20] proved that existence of $\phi$-recurrent $(LCS)_3$ manifold which is neither locally symmetric nor locally $\phi$-symmetric by non trivial examples. Later, G.T. Shreenivasa, Venkatesha, C.S. Bagewadi [23] also studied $(LCS)_{2n+1}$-manifolds in 2009. Further,in 2011 Atceken and Hui have been studied on the submanifold of an $(LCS)_n$. The $(LCS)$ manifolds have been also studied by Atceken [14], Narain and Yadav [15], Prakasha [16], Shaikh [17],Shaikh et al. [14,18],Shaikh and Bihn [21], shaikh and Hui [21], Sreenivasa et al. [22], Yadav et al. [23] and others. Recently, in 2016 Shaikh, Matsuyama and Hui have been studied on invariant submanifolds of $(LCS)_n$-manifolds and it deals with the study of some basic properties of invariant submanifolds of $(LCS)_n$-manifolds.

If $\bar{\nabla}$ denotes semi-symmetric metric connection on a contact metric manifold, then it is given by [13]

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi, \tag{1.1}$$

Where $\eta(Y) = g(Y,\xi)$.

The covariant differential of the $p^{th}$ order, $p \geq 1$, of a (0, $k$)-tensor field $T$, $k \geq 1$, defined on a Riemannian manifold $(M,g)$ with the Levi-civita connection $\nabla$, is denoted by $\nabla^P T$.

The tensor $T$ is said to be recurrent if it satisfies

$$(\nabla T)(X_1, \ldots .X_K; X)T(Y_1, \ldots .Y_K) = (\nabla T)(Y_1, \ldots .Y_K; X)T(X_1, \ldots .X_K), \tag{1.2}$$

On M. where $X, Y, X_1, Y_1, \ldots, X_K, Y_K \in TM$. From (1.2) it follows that at a point $x \in M$, if the tensor $T$ is non zero, then there exist a unique 1-form $\phi$ and a (0, 2)-tensor $\psi$, definedon a neighbourhood $U$ of $x$ such that

$$\nabla T = T \otimes \phi, \quad \phi = d(log\|T\|). \tag{1.3}$$

Similarly,the tensor $T$ is said to be 2-recurrent [13] if it satisfies

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$(\nabla^2 T)(X_1, \dots .X_K; X, Y)T(Y_1, \dots .Y_K) =$
$(\nabla^2 T)(Y_1, \dots .Y_K; X, Y)T(X_1, \dots .X_K),$ (1.4)

On M. Where $X, Y, X_1, Y_1, \dots, X_K, Y_K \in TM$. Now from (1.4) it follows that at a point $x \in M$, if the tensor $T$ is non zero, then there exist a (0, 2)-tensor $\psi$,defined on a neighborhood $U$ of x such that

$\nabla^2 T = T \otimes \psi,$ (1.5)

hold on $U$ , where $\| T \|$ denotes the norm of $T$ and

$$\| T \|^2 = g(T, T).$$

The tensor $T$ is said to be generalized 2-recurrent if

$$((\nabla^2 T)(X_1, \dots .X_K; X, Y) - (\nabla T \otimes \phi)(X_1, \dots .X_K; X, Y))T(Y_1, \dots .,Y_K) = ((\nabla^2 T)(Y_1, \dots .Y_K; X, Y) - (\nabla T \otimes \phi)(Y_1, \dots .Y_K; X, Y))T(X_1, \dots .,X_K),$$
(1.6)

hold on $M$ , where $\phi$ is a 1-form on $M$ . From (1.6) it follows that at a point $x \in M$ if the tensor $T$ is non zero, then there exists a (0, 2)-tensor $\psi$,defined on a neighborhood $U$ of $x$ such that

$\nabla^2 T = \nabla T \otimes \phi + T \otimes \psi,$ (1.7)

holds on $U$.

## II. ISOMETRIC IMMERSION

Let $f : (M, g) \to (\tilde{M}, \tilde{g})$ be an isometric immersion from an $n$-dimensional Riemannian manifold $(M, g)$ into $(n+d)$-dimensional Riemannian manifold $(\tilde{M}, \tilde{g})$, $n \geq 2$, $d \geq 1$.We denote by $\nabla$ and $\tilde{\nabla}$ as Levi-Civita connection of $M^n$ and $\tilde{M}^{n+d}$ respectively. Then the formulas of Gauss and Weingarten are given by

$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y),$ (2.1)

$\tilde{\nabla}_X N = -A_N X + \nabla_X^{\perp} N,$ (2.2)

for any tangent vector fields $X$, $Y$ and the normal vector field $N$ on $M$ . Where

$\sigma$ -second fundamental form,

$A$-the shape operator,

$\nabla^{\perp}$-is the normal connection.

If the second fundamental form $\sigma$ is identically zero, then the manifold is said to be totally geodesic. The second fundamental form $\sigma$ and $A_N$ are related by

$\tilde{g}(\sigma(X, Y), N) = g(A_N X, Y),$ (2.3)

for tangent vector fields $X$, $Y$.

The first and second covariant derivatives of the second fundamental form $\sigma$ are given by

$$(\tilde{\nabla}_X \sigma)(Y, Z) = \nabla_X^{\perp}(\sigma(Y, Z)) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z),$$
(2.4)

$$(\tilde{\nabla}^2 \sigma)(Z, W, X, Y) = (\tilde{\nabla}_X \tilde{\nabla}_Y \sigma)(Z, W),$$
$$= \nabla_X^{\perp}((\tilde{\nabla}_Y \sigma)(Z, W)) - (\tilde{\nabla}_Y \sigma)(\nabla_X Z, W) - (\tilde{\nabla}_X \sigma)(Z, \nabla_Y W) - (\tilde{\nabla}_{\nabla_X Y} \sigma)(Z, W),$$
(2.5)

respectively, where $\tilde{\nabla}$ is called the Vander Waerden-Bortolotti connection of $M$ [7].

If

$$\tilde{\nabla}\sigma = 0$$

then $M$ is said to have parallel second fundamental form [7].

## III. LCS-MANIFOLD

In 2003 A. A Shaikh the first author introduced the notion of Lorentzian concircular struc- ture manifold(briefly, $(LCS)_n$-manifolds) and these are applied in General theory ofrelativity and cosmology.

An n-dimensional Lorentzian Manifold $M$ is a smooth connected para-compact Housdorff manifold with a Lorentzian metric $g$. that is, $M$ admits a smooth symmetric tensor field $g$ of type (0, 2) such that for each point $p \in M$ , the tensor $g_p : T_pM \times T_pM \to R$ is a non-degenerate inner product of signature (−, +, ....+), where $T_pM$ denotes the tangent vector space of $M$ at $p$ and $R$ is the real number space. A non-zero vector $v \in T_pM$ is said to be timelike if it satisfies $g_p(v, v) < 0$. The category to which a given vector fields is called its casual character.

In a Lorentzian manifold $(M, g)$,a vector field $P$ defined by

$g(X, P) = A(X),$

for any $X$ on $M$ , is said to be a concircular vector field if

$(\nabla_X A)(Y) = \alpha\{g(X, Y) + \omega(X)A(Y)\},$ (3.1)

for $Y$ on $M$ ,where $\alpha$-is a non-zero scalar and $\omega$-is a closed 1-form.

Let $M$ be Lorentzian manifold admitting a unit timelike concircular vector field $\xi$, called the structure vector field of the manifold. Then we have

$g(\xi, \xi) = -1,$ (3.2)

Since $\xi$ is a unit concircular vector field, it follows that there exist a non-zero 1-form $\eta$ such that

$g(X, \xi) = \eta(X),$ (3.3)

The following equation $(\nabla_X \eta)(Y) = \alpha\{g(X, Y) + \eta(X)\eta(Y)\},$ (3.4)

holds for all vector fields $X$, $Y$ on $M$ and $\alpha$ is a non-zero scalar function satisfies

$\nabla_X \alpha = X(\alpha) = d\alpha(X) = \rho\eta(X),$ (3.5)

$\rho$ being a certain scalar function given by

$\rho = -\xi(\alpha),$ (3.6)


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If we put

$\nabla_X \xi = \alpha \varphi X,$ (3.7)

Then from (3.4) and (3.7) we have

$\varphi X = X + \eta(X)\xi,$ (3.8)

From which it follows that

$\varphi^2 = X + \eta(X)\xi,$ (3.9)

that is, $\varphi$ is a symmetric (1,1) tensor field, called the structure tensor of the manifold. The n-dimensional Lorentzian manifold $M$ together with the unit timelike concircular vector field $\xi$, its associated 1-form $\eta$, and an (1, 1) tensor field $\varphi$ is said to be a Lorentzian concircular structure manifold (briefly, $(LCS)_n$-manifold). Especially, if $\alpha$=1, then we can obtain the LP-Sasakian structure of Matsumoto.

In $(LCS)_n$-manifold, the following results hold

$\eta(\xi) = g(\xi,\xi) = -1,\ \phi^2 X = X + \eta(X)\xi,\ g(X,\xi) = \eta(X), \phi\xi = 0, \eta o \phi = 0.$ (3.10)

$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y),$ (3.11)

$\eta(R(X,Y)Z) = (\alpha^2 - \rho)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\},$ (3.12)

$S(X,\xi) = (\alpha^2 - \rho)(n-1)\eta(X),$ (3.13)

for any vector fields $X$, $Y$ , $Z$ on $M$ and $\alpha^2 - \rho$ $f$= 0, where $R$-curvature tensor and $S$-Ricci tensor of the manifold.

In an $(LCS)_n$-manifold, we also have the following results

$(\nabla_X \eta)(Y) = (\nabla_Y \eta)(X)$ (3.14)

$d\eta(X,Y) = 0$ (3.15)

We also mention that, in an $(LCS)_n$-manifold the symmetric (1, 1) tensor field $\phi$ is idem- potent and hence the eigen value of $\phi$ is either 1 or 0.

## IV. INARIANT SUBMANIFOLDS OF (LCS)-MANIFOLD ADMITTING SEMI-SYMMETRIC METRIC CONNECTION

If $\tilde{M}$ is a $(LCS)$-manifold with structure tensors $(\bar{\phi},\bar{\xi},\bar{\eta},\bar{g})$ then we know that its invariant submanifold $M$ has the induced $(LCS)$ -structure $(\phi,\xi,\eta,g)$.

A submanifold $M$ of a $(LCS)$-manifold $\tilde{M}$ with a semi-symmetric metric connection is called an invariant submanifold of $\tilde{M}$ with a semi- symmetric connection, if for each $x \epsilon M$, $\phi(T_x M) \subset T_x M$. The study of such submanifolds for the different type of contact manifolds have been carried out by authors of [4, 12], [26]-, [29], [33, 37]

As a consequence, $\xi$ becomes tangent to M . For an invariant submanifold of a (LCS) manifold with a semi-symmetric metric connection, We have

$\sigma(X,\xi) = 0,$ (4.1)

for any vector $X$ tangent to $M$ .

Let $\bar{M}$ be a $(LCS)$-manifold admitting a semi-symmetric metric connection $\tilde{\nabla}$ ·

**Lemma 4.1.** *Let $M$ be an invariant submanifold of contact metric manifold $\tilde{M}$ which admits semi-symmetric connection $\bar{\tilde{\nabla}}$ and let $\sigma$ and $\bar{\sigma}$ be the second fundamental form with respect to Levi-civita connection and* semi-symmetric metric connection then

**(a)** M admits semi-symmetric metric connection,

**(b)** the second fundamental form with respect to $\tilde{\nabla}$ and $\bar{\tilde{\nabla}}$ are equal.

*Proof. :* we know that the contact metric structure $(\bar{\phi},\bar{\xi},\bar{\eta},\bar{g})$ on $\tilde{M}$ induces $(\phi,\xi,\eta,g)$ on invariant submanifold. By virtue of (1.1) , we get

$\bar{\tilde{\nabla}}_X Y = \tilde{\nabla}_X Y + \eta(Y)X - g(X,Y)\xi.$ (4.2)

By using (2.1) in (4.2)

$\bar{\tilde{\nabla}}_X Y = \nabla_X Y + \sigma(X,Y) + \eta(Y)X - g(X,Y)\xi.$ (4.3)

Now gauss formula (2.1) with respect to semi-symmetric metric connection is given by

$\bar{\tilde{\nabla}}_X Y = \bar{\nabla}_X Y + \bar{\sigma}(X,Y).$ (4.4)

Equating (4.3) and (4.4), we get (1.1)

$\bar{\sigma}(X,Y) = \sigma(X,Y).$ (4.5)

## V. RECURRENT INARIANT SUBMANIFOLDS OF (LCS)-MANIFOLD ADMITTING SEMI-SYMMETRIC METRIC CONNECTION

We consider invariant submanifolds of a $(LCS)$-manifold When $\sigma$ is recurrent, 2-recurrent,generalized 2-recurrent and $M$ has parallel third fundamental form with respect to semi-symmetric metric connection. We write the equations (2.4)and (2.5) with respect to semi-symm etric metric connection in the form

$(\bar{\tilde{\nabla}}_X \sigma)(Y,Z) = \bar{\nabla}_X^{\perp}(\sigma(Y,Z)) - \sigma(\bar{\nabla}_X Y, Z) - \sigma(Y, \bar{\nabla}_X Z),$ (5.1)

$(\bar{\tilde{\nabla}}_X^2)(Z,W,X,Y) = (\bar{\tilde{\nabla}}_X \bar{\tilde{\nabla}}_Y \sigma)(Z,W) = \bar{\nabla}_X^{\perp}\left((\bar{\tilde{\nabla}}_Y \sigma)(Z,W)\right) - (\bar{\tilde{\nabla}}_Y \sigma)(\bar{\nabla}_X Z, W) - (\bar{\tilde{\nabla}}_X \sigma)(Z, \bar{\nabla}_Y W) - (\bar{\tilde{\nabla}}_{\bar{\nabla}_X Y} \sigma)(Z,W).$ (5.2)

**Theorem 5.1.** *:Let $M$ be an invariant submanifold of $(LCS)_n$-manifold $\tilde{M}$admitting semi-symmetric metric connection. Then $\sigma$ is recurrent with respect to semi-symmetric metric connection if and only if it is totally geodesic with respect to Levi-Civita connection.*

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***Proof.*** :Let $\sigma$ be recurrent with respect to semi-symmetric metric connection. Then from (1.3) we get

$(\bar{\bar{\nabla}}_X\sigma)(Y,Z) = \phi(X)\,\sigma(Y,Z),$

Where $\phi$ is a 1-form on M. By using (5.1) and put $Z=\xi$ in the above equation, we have

$$\bar{\nabla}_X^{\perp}\big(\sigma(Y,\xi)-\sigma(\bar{\nabla}_X Y,\xi)-\sigma(Y,\bar{\nabla}_X\xi)\big) = \phi(X)\,\sigma(Y,\xi),$$

Which by virtue of (4.1) reduces to

$$-\sigma(\bar{\nabla}_X Y,\xi)-\sigma(Y,\bar{\nabla}_X\xi)=0. \quad (5.4)$$

Using (1.1), (3.7),(3.10) and (4.1) in (5.4)

$$-\alpha\sigma(Y,\phi X)+\sigma(Y,X)=0. \quad (5.5)$$

ow replace $X$ by $\phi X$ and by virtue of (3.10), (4.1) in (5.5), we get

$$(1-\alpha)\,\sigma(Y,X)=0. \quad (5.6)$$

We get $\sigma(X,Y)=0$, if $\alpha\neq 1$ Therefore $M$ is totally geodesic provided $\alpha\neq 1$

**Theorem 5.2.** *Let M be an invariant submanifold of a (LCS)-manifold $\tilde{M}$ admitting semi-symmetric metric connection. Then M has parallel third fundamental form with respect to semi-symmetric metric connection if and only if it is totally geodesic with respect to Levi- Civita connection.*

*Proof.* Let $M$ has parallel third fundamental form with respect to semi-symmetric metric connection. then we have

$(\bar{\bar{\nabla}}_X\bar{\bar{\nabla}}_Y\sigma)(Z,W)=0$

Taking W=$\xi$ and using (5.2)in the above equation,We have

$$\bar{\nabla}_X^{\perp}\big((\bar{\bar{\nabla}}_Y\sigma)(Z,\xi)\big)-(\bar{\bar{\nabla}}_Y\sigma)(\bar{\nabla}_X Z,\xi)-(\bar{\bar{\nabla}}_X\sigma)(Z,\bar{\nabla}_Y\xi)-(\bar{\bar{\nabla}}_{\bar{\nabla}_X Y}\sigma)(Z,\xi)=0. \quad (5.7)$$

$$0=\bar{\nabla}_X^{\perp}\{\sigma(\bar{\nabla}_Y Z,\xi)+\sigma(Z,\ \bar{\nabla}_Y\xi)\}-\bar{\nabla}_Y^{\perp}\big(\sigma(\bar{\nabla}_X Z,\xi)\big)+\sigma(\bar{\nabla}_Y\bar{\nabla}_X Z,\xi)+2\sigma(\bar{\nabla}_X Z,\ \bar{\nabla}_Y\xi)-\bar{\nabla}_X^{\perp}\big(\sigma(Z,\ \bar{\nabla}_Y\xi)\big)+\sigma(Z,\bar{\nabla}_X\bar{\nabla}_Y\xi)+\sigma\big(\bar{\nabla}_{\bar{\nabla}_X Y}\xi\big). \quad (5.8)$$

In view of (1.1),(3.7),(3.10) and (4.1) the above result (5.8) gives

$$0=-2\bar{\nabla}_X^{\perp}\sigma(Z,\alpha\phi Y)+2\bar{\nabla}_X^{\perp}\sigma(Z,Y)+\sigma(Z,\nabla_X\alpha\phi Y)-\sigma(Z,\nabla_X\eta(Y)\xi)+\alpha\sigma(Z,\phi\nabla_X Y)+\alpha\eta(Y)\sigma(Z,\phi X)-\alpha\sigma(Z,\nabla_X Y)-\eta(Y)\sigma(Z,X)+2\alpha\sigma(\nabla_X Z,\phi Y)-2\sigma(\nabla_X Z,Y)+2\alpha\eta(Z)\sigma(X,\phi Y)-2\eta(Z)\sigma(X,Y). \quad (5.9)$$

Put Y= $\xi$ and using(3.7),(3.10) and (4.1) in (5.9),We get

$$0=(\alpha^2+1)\sigma(Z,X)-2\alpha\sigma(Z,\phi X). \quad (5.10)$$

Multiply by $(\alpha^2+1)$ in (5.10), we get

$$0=(\alpha^2+1)^2\sigma(Z,X)-2\alpha(\alpha^2+1)\sigma(Z,\phi X). \quad (5.11)$$

Put $X=\phi X$ in (5.10), we get

$$0=(\alpha^2+1)\sigma(Z,\phi X)-2\alpha\sigma(Z,X). \quad (5.12)$$

Multiply by $2\alpha$ in (5.12) , we get

$$0=2\alpha(\alpha^2+1)\sigma(Z,\phi X)-4\alpha^2\sigma(Z,X). \quad (5.13)$$

(5.3)

Adding equation (5.11) and (5.13), we get $\alpha\neq\pm1$ and $\sigma(X,Z)=0$.Thus $M$ is totally geodesic. The converse statement is trivial. This proves the theorem

**Corollary 5.1.** *:Let M be an invariant submanifold of a (LCS)-manifold $\tilde{M}$ admitting semi-symmetric metric connection. Then $\sigma$ is 2-recurrent with respect to semi-symmetric metric connection if and only if it is totally geodesic with respect to Levi- Civita connection.*

***Proof.*** Let $\sigma$ be 2-recurrent with respect to semi-symmetric metric connection. From (1.5), We have

$(\bar{\bar{\nabla}}_X\bar{\bar{\nabla}}_Y\sigma)(Z,W)=\sigma(Z,W)\phi(X,Y).$

Taking W=$\xi$ and using (5.2) in the above equation, We have

$$\bar{\nabla}_X^{\perp}\big((\bar{\bar{\nabla}}_Y\sigma)(Z,\xi)\big)-(\bar{\bar{\nabla}}_Y\sigma)(\bar{\nabla}_X Z,\xi)-(\bar{\bar{\nabla}}_X\sigma)(Z,\bar{\nabla}_Y\xi)-(\bar{\bar{\nabla}}_{\bar{\nabla}_X Y}\sigma)(Z,\xi)=\sigma(Z,\xi)\phi(X,Y). \quad (5.14)$$

In view of (4.1) and (5.1) We write (5.14) in the form

$$0=\bar{\nabla}_X^{\perp}\{\sigma(\bar{\nabla}_Y Z,\xi)+\sigma(Z,\ \bar{\nabla}_Y\xi)\}-\bar{\nabla}_Y^{\perp}\big(\sigma(\bar{\nabla}_X Z,\xi)\big)+\sigma(\bar{\nabla}_Y\bar{\nabla}_X Z,\xi)+2\sigma(\bar{\nabla}_X Z,\ \bar{\nabla}_Y\xi)-\bar{\nabla}_X^{\perp}\big(\sigma(Z,\ \bar{\nabla}_Y\xi)\big)+\sigma(Z,\bar{\nabla}_X\bar{\nabla}_Y\xi)+\sigma\big(\bar{\nabla}_{\bar{\nabla}_X Y}Z,\xi\big)+\sigma\big(Z,\bar{\nabla}_{\bar{\nabla}_X Y}\xi\big).$$

(5.15)

Using (1.1),(3.7),(3.10) and (4.1) in (5.15), We get

$$0=-2\bar{\nabla}_X^{\perp}\sigma(Z,\alpha\phi Y)+2\bar{\nabla}_X^{\perp}\sigma(Z,Y)+\sigma(Z,\nabla_X\alpha\phi Y)-\sigma(Z,\nabla_X\eta(Y)\xi)+\alpha\sigma(Z,\phi\nabla_X Y)+\alpha\eta(Y)\sigma(Z,\phi X)-\alpha\sigma(Z,\nabla_X Y)-\eta(Y)\sigma(Z,X)+2\alpha\sigma(\nabla_X Z,\phi Y)-2\sigma(\nabla_X Z,Y)+2\alpha\eta(Z)\sigma(X,\phi Y)-2\eta(Z)\sigma(X,Y).$$

(5.16)

Put $Y=\xi$ and using (3.7),(3.10)and(4.1) in (5.16),We get

$$0=(\alpha^2+1)\sigma(Z,X)-2\alpha\sigma(Z,\phi X) \quad (5.17)$$

Multiply by $(\alpha^2+1)$ in (5.17), we get

$$0=(\alpha^2+1)^2\sigma(Z,X)-2\alpha(\alpha^2+1)\sigma(Z,\phi X) \quad (5.18)$$

Put $X=\phi X$ in (5.17), we get

$$0=(\alpha^2+1)\sigma(Z,\phi X)-2\alpha\sigma(Z,X) \quad (5.19)$$

Multiply by $2\alpha$ in (5.19), we get

$$0=2\alpha(\alpha^2+1)\sigma(Z,\phi X)-4\alpha^2\sigma(Z,X) \quad (5.20)$$


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Adding equation (5.18) and (5.20), we get $\alpha \neq \pm 1$ and $\sigma(X,Z) = 0$.Thus $M$ is totally geodesic. The converse statement is trivial. This proves the theorem.

**Theorem 5.3.** *:Let $M$ be an invariant submanifold of a $(LCS)$-manifold $\tilde{M}$ admitting semi-symmetric metric connection. Then $\sigma$ is genaralised 2-recurrent with respect to semi-symmetric metric connection if and only if it is totally geodesic with respect to Levi- Civita connection.*

*Proof.* Let $\sigma$ be a generalized 2-recurrent with respect to semi-symmetric metric connection.

From (1.7), We have

$$(\bar{\bar{\nabla}}_X \bar{\bar{\nabla}}_Y \sigma)(Z,W) = \psi(X,Y)\,\sigma(Z,W) + \phi(X)(\bar{\bar{\nabla}}_Y \sigma)(Z,W). \tag{5.21}$$

Where $\psi$ and $\phi$ are 2-recurrent and 1-form respectively. Taking $W = \xi$ in (5.21)and using (4.1),We get

$(\bar{\bar{\nabla}}_X \bar{\bar{\nabla}}_Y \sigma)(Z,\xi) = \phi(X)(\bar{\bar{\nabla}}_Y \sigma)(Z,\xi)$.

Using (4.1),(5.1)and(5.2) in above equation, We get

$$\bar{\nabla}_X^{\perp}\left((\bar{\bar{\nabla}}_Y \sigma)(Z,\xi)\right) - (\bar{\bar{\nabla}}_Y \sigma)(\bar{\nabla}_X Z,\xi) - (\bar{\bar{\nabla}}_X \sigma)(Z,\bar{\nabla}_Y \xi) - (\bar{\bar{\nabla}}_{\bar{\nabla}_X Y}\sigma)(Z,\xi) = -\phi(X)\{\sigma(\bar{\nabla}_Y Z,\xi) + \sigma(Z,\bar{\nabla}_Y \xi)\} \tag{5.22}$$

By using (4.1) and by the virtue of (5.1), We write (5.22) in the form

$$\bar{\nabla}_X^{\perp}\{\sigma(\bar{\nabla}_Y Z,\xi) + \sigma(Z,\bar{\nabla}_Y \xi)\} - \bar{\nabla}_Y^{\perp}(\sigma(\bar{\nabla}_X Z,\xi)) + \sigma(\bar{\nabla}_Y \bar{\nabla}_X Z,\xi) + 2\sigma(\bar{\nabla}_X Z,\bar{\nabla}_Y \xi) - \bar{\nabla}_X^{\perp}(\sigma(Z,\bar{\nabla}_Y \xi)) + \sigma(Z,\bar{\nabla}_X \bar{\nabla}_Y \xi) + \sigma(\bar{\nabla}_{\bar{\nabla}_X Y} Z,\xi) + \sigma(Z,\bar{\nabla}_{\bar{\nabla}_X Y}\xi) = -\phi(X)\{\sigma(\bar{\nabla}_Y Z,\xi) + \sigma(Z,\bar{\nabla}_Y \xi)\}. \tag{5.23}$$

In view of (1.1),(3.7),(3.10) and (4.1) in (5.23), We get

$$-2\bar{\nabla}_X^{\perp}\sigma(Z,\alpha\phi Y) + 2\bar{\nabla}_X^{\perp}\sigma(Z,Y) + \sigma(Z,\nabla_X \alpha\phi Y) - \sigma(Z,\nabla_X \eta(Y)\xi) + \alpha\sigma(Z,\phi\nabla_X Y) + \alpha\eta(Y)\sigma(Z,\phi X) - \alpha\sigma(Z,\nabla_X Y) - \eta(Y)\sigma(Z,X) + 2\alpha\sigma(\nabla_X Z,\phi Y) - 2\sigma(\nabla_X Z,Y) + 2\alpha\eta(Z)\sigma(X,\phi Y) - 2\eta(Z)\sigma(X,Y) = -\phi(X)\{\sigma(Z,\alpha\phi Y) - \sigma(Z,Y)\}. \tag{5.24}$$

Put $Y = \xi$ and using(3.7),(3.10) and (4.1) in (5.24),We get

$$0 = (\alpha^2+1)\sigma(Z,X) - 2\alpha\sigma(Z,\phi X). \tag{5.25}$$

Multiply by $(\alpha^2+1)$ in (5.25), we get

$$0 = (\alpha^2+1)^2\sigma(Z,X) - 2\alpha(\alpha^2+1)\sigma(Z,\phi X). \tag{5.26}$$

Put $X = \phi X$ in (5.25), we get

$$0 = (\alpha^2+1)\sigma(Z,\phi X) - 2\alpha\sigma(Z,X). \tag{5.27}$$

Multiply by $2\alpha$ in (5.27), we get

$$0 = 2\alpha(\alpha^2+1)\sigma(Z,\phi X) - 4\alpha^2\sigma(Z,X). \tag{5.28}$$

Adding equation (5.26) and (5.28), we get $\alpha \neq \pm 1$ and $\sigma(X,Z) = 0$.Thus $M$ is totally geodesic. The converse statement is trivial. This proves the theorem.

## VI. Conclusion

From the Theorems (5.1), (5.2) and (5.3) and from the Corollary (5.1), we conclude that:

If $M$ is an invariant submanifolds of LCS manifold admitting semi-symmetric metric connection then the following conditions are equivalent:

i. The submanifold M is totally geodesic with respect to the Levi-Civita connection;
ii. Second fundamental form σ is recurrent with respect to semi-symmetric metric connection;
iii. The submanifold M has parallel third fundamental form with respect to semi-symmetric metric connection;
iv. Second fundamental form σ is 2-recurrent with respect to semi-symmetric metric connection;
v. Second fundamental form σ is generalized 2-recurrent with respect to semi-symmetric metric connection.

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