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Analysis and graphical representation with electronic circuit for reciprocating motion by electromagnetic and spring forces

Ankit R. Sharma†, Aditya V. Natu‡

†Department of Mechanical Engineering, Savitribai Phule Pune University, Pune, India

‡ Department of Mechanical Engineering, Savitribai Phule Pune University, Pune, India

Abstract

An electromagnet can be set into oscillatory motion by using electromagnets powered by Alternating Current of low frequency, which then slides on a frame and can be used to rotate a crank. The Alternating current makes the electro-magnets switch poles — both the oscillating poles being out of phase — causing the oscillations. An oscillator is used to generate the Alternating current of desired frequency. A differential equation is derived by drawing the free-body-diagram of the bar magnet to analyze the motion of the oscillating permanent magnet, whose graph has been plotted after giving relevant initial conditions (using Wolfram Alpha Software), which are necessary for the proper working of the mechanism explained below. It is observed that a motion extremely close to that of an SHM is obtained for the bar magnet.

Keywords: Oscillatory motion, Reciprocating motion, Oscillator Circuit, Electromagnets, Springs, Bar Magnet

1. Introduction

In this mechanism, we have used a permanent bar magnet and two electro magnets. This mechanism is used to simulate reciprocating motion. It is known that in magnets, opposite poles attract while similar poles repel. Thus, when the north pole of the bar magnet faces the north pole of the electro-magnet, and the south pole of the bar magnet faces the south pole of the other electromagnet, the bar magnet gets repelled from both the sides and is forced to move to the center. This is called stable equilibrium. The opposite happens when the north pole of the bar magnet faces the electro-magnet south pole and the bar magnet south pole, the electromagnet north pole. The bar magnet is attracted from both the sides. This is called unstable equilibrium.

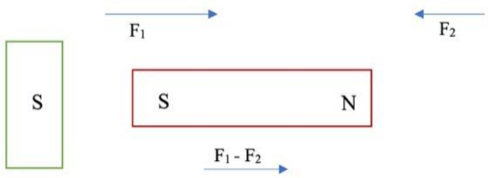

Fig.1 Stable equilibrium phase of mechanism—Free Body Diagram (FBD).

In the above FBD, the green-outlined boxes represent electromagnets, while the red-outlined box is the permanent bar magnet. The electro-magnets are at fixed positions, whereas the bar magnet is free to slide along the line joining the poles of the two electro-magnets.

Here, F_1 is the force exerted by the pole closer to the bar magnet, while force F_2 is the force exerted by the other pole, which is at a greater distance from the magnet. We know that magnetic forces get stronger as the poles move closer. Hence, $F_1 > F_2$, meaning that the bar magnet will be pushed in the direction of F_1, i.e. away from the electro-magnet south pole and towards the center.

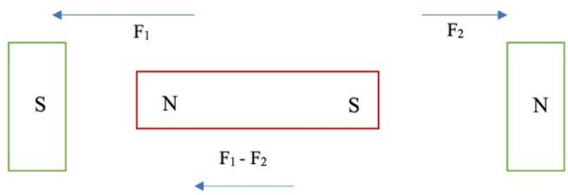

Fig.2 Unstable equilibrium phase of mechanism—Free body Diagram

Here, F_2 is the force exerted by the pole closer to the bar magnet, and force F_1 is the force exerted by the other pole, which is at a greater distance from the magnet. We know that magnetic forces get weaker as the poles move further. Hence, $F_1 > F_2$, meaning that the bar magnet will be pushed in the direction of F_1, i.e. towards the electro-magnet south pole and away from the center.

2. Construction of main mechanism

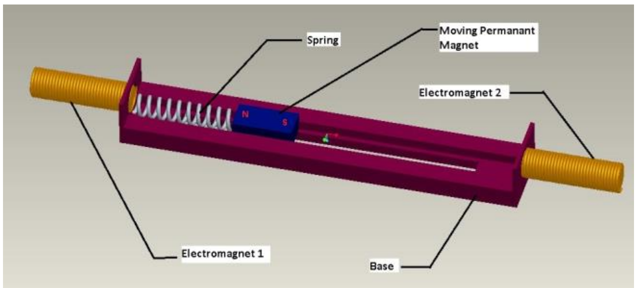

Fig.3 The construction of the reciprocating mechanism.

A frame or base (Represented by Maroon in Fig.3) with a long slot is constructed and the bar magnet (shown in blue in Fig. 3) is placed over it. The electromagnets (represented in yellow in Fig. 3), shown with the copper coils are placed on either ends of the frame. Springs are connected to the permanent magnet and joined to either ends of the frame. The springs serve two purposes:

i. To prevent the permanent magnet from moving too close to either electromagnet.
ii. To provide an additional force in case one magnetic force becomes too weak due to distance between poles getting too large.
iii. To prevent bar magnet from locking at a stable position which can cease it's motion due to locking action.

The coils are supplied Alternating Current of desired frequency using an electronic circuit explained below.

3. Construction of Electronic Circuit

In order to produce alternating magnetic fields in the electromagnets, the electromagnets need to be supplied an Alternating Current. The frequency of the alternating magnetic fields is the same as that of the AC supply. This frequency is equal to the frequency of the reciprocating motion. A high voltage and low frequency of the AC is desirable.

High frequency AC might lead to a slip — the movable magnet may not reciprocate at the synchronous speed. High frequency AC may also give rise to severe heating due to friction. We need to arrange for a low frequency and high voltage AC supply. AC Mains supply cannot be used directly, it being of the frequency 50 Hz.

This can be achieved by two means
1. By using a signal generator

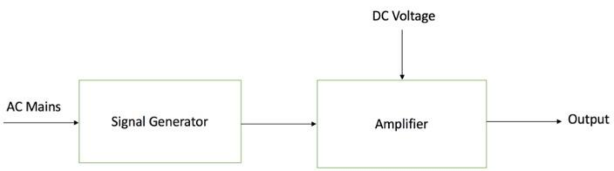

Fig.4 The construction of the electronic circuit using signal generator.

A signal generator is a device capable of producing an alternating current of any desired frequency. The output voltage of the signal generator is typically less than 20V. Therefore, we need an amplifier to amplify it. The amplifier used is an Operational Amplifier (IC LTC6090, OPA454, OPA549, OPA547, LM143, LM741, LM340, LM120, LM108, etc.). The Operational Amplifier (Op-Amp) needs a DC dual power supply, which has to be provided. The DC supply needed is ±20V for LM741 and for ±40V LM143.

Pros:
1. Easy to construct
2. Less complicated
3. Easy to operate

Cons:
1. Heavier
2. Bulkier
3. Requires 230 V power supply
4. Highly expensive

2. By using an oscillator.

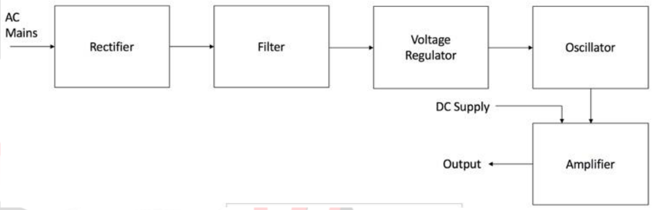

Fig.5 The construction of the electronic circuit without using signal generator.

This circuit is used to first convert the AC from Mains to DC and again from DC to AC of desired frequency. This circuit is inexpensive as no signal generator is required. The entire circuit can be built on a Printed Circuit Board (PCB), which makes it compact. However, building this circuit takes time. Also, the components used should be carefully selected to ensure that the circuit produces the required output but does get damaged.

•The rectifier makes the bidirectional AC unidirectional.
•The filter circuit is used to make the output of the rectifier steady.
•The voltage regulator is used to produce a constant voltage.
•The Oscillator is used to convert DC into AC. The Oscillator used here is the Colpitt's Oscillator.
•The amplifier amplifies the output of the oscillator. An Op-Amp is used as a non-inverting amplifier.

Pros:
1. Cheaper
2. Can be operated on a significantly low voltage
3. Lighter
4. Portable

Cons:
1. Difficult to Construct
2. Complex design

4. Mathematical Analysis

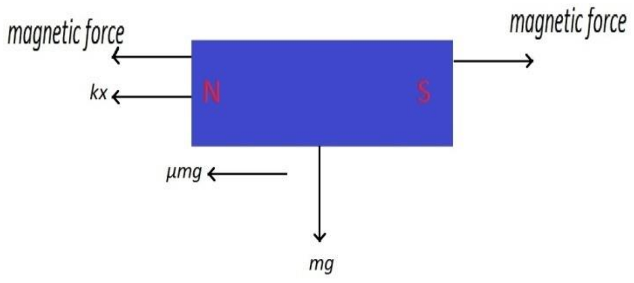

Fig.5 The Free Body Diagram of the permanent bar magnet.

The following nomenclature shall be used in this section:

Table 1 Nomenclature used in this section

Parameter	Symbol
Spring constant	k
Position	x
Time	t
Mass of bar magnet	m
Coefficient of friction	μ
Permeability of air	μ_0
Distance between electromagnetic pole and bar magnet pole	r
Initial distance between the electromagnet and bar magnet poles	l
Pole strength of electromagnet	M_1
Pole strength of bar magnet	M_2
Angular speed of oscillations	ω

The spring force on the movable magnet is

$$F = -kx$$

Where, k represents the spring constant and x is the deformation of the spring.

The force between a pole of the bar magnet and a pole of an electromagnet is

$$F = \frac{\mu_o M_1 M_2}{4\pi r^2} = \frac{J}{r^2}$$

The electromagnet is powered by an Alternating Current. As a result, the pole strength of the electromagnetic pole varies sinusoidally and that of the permanent magnet remains constant. Then, the force between the poles also varies sinusoidally, as given below

$$F = \frac{\mu_o M_1 M_2 \cos(\omega t)}{4\pi r^2}$$

Where M_1 is the amplitude of the varying pole strength of the electromagnet

Summing up the forces on the bar magnet as seen in the FBD and applying the Newton's second law of motion, we get.

$$ma = \Sigma F$$

Where a is the acceleration, $a = \frac{d^2x}{dt^2}$

By substituting the values of the forces, we get

$$m\frac{d^2x}{dt^2} = -kx - \mu mg - J\cos(\omega t)\left(\frac{1}{(l+x)^2} - \frac{1}{(l-x)^2}\right)$$

or

$$m\frac{d^2x}{dt^2} + kx + \mu mg + J\cos(\omega t)\left(\frac{1}{(l+x)^2} - \frac{1}{(l-x)^2}\right) = 0$$

The complementary function of the differential equation is

$$y = C_1\sin(\omega t) + C_2\cos(\omega t)$$

Where C_1 and C_2 are constants and $\omega = \sqrt{\frac{k}{m}}$

Using this equation, we can calculate the natural frequency of the bar magnet-spring system. It is desirable to keep the frequency of the Alternating current a as close as possible to this frequency for resonance to occur which gives high amplitude oscillations.

5. Graphical Analysis

In order to plot the following g graphs, following values were selected

Table 2 Parameters selected while solving the differential equation

Parameter	Value
Mass of bar magnet	100 grams
Length of bar magnet	5 cm
Total length of the slot	20 cm
Pole strength	8 A.m
Frequency of AC	5 Hz
Coefficient of friction	0.1

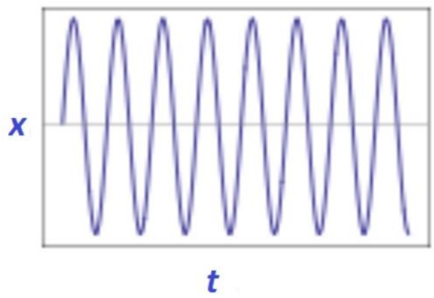

Fig.6 The Position vs. Time plot for the bar magnet obtained from the differential equations derived in mathematical analysis section.

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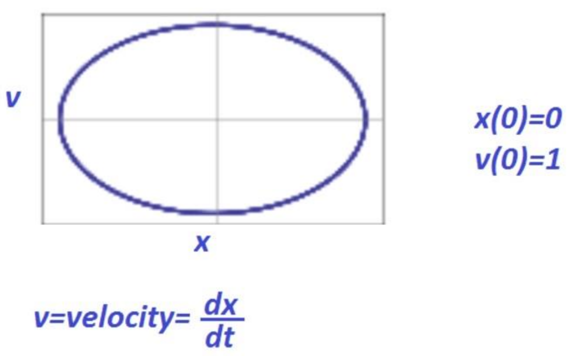

Fig.7 The Velocity vs. Position plot for the bar magnet obtained from the differential equations derived in mathematical analysis section. The initial velocity is assumed to be non-zero while the initial position is taken to be at the equilibrium point.

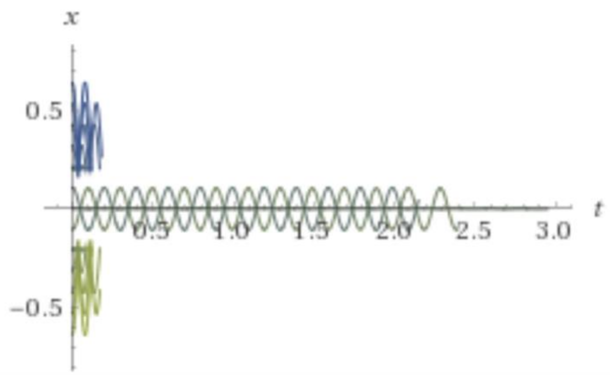

Fig.8 The graph of family of solutions of position vs. time plotted for obtaining maximum and minimum values of position of bar magnet.

6. Results

From the above graphs, we get the following results:

1. From Fig. 6, we observe that the Position vs. Time plot for the mechanism represents a reciprocating motion which closely approximates to a Simple Harmonic Motion

2. From Fig. 7, we find that the velocity is highest at the equilibrium point and is zero at extreme points, which clearly depicts and reciprocating motion.

3. From Fig. 8, we are able to determine the family of solutions for which the mechanism would work as near-SHM type reciprocating motion. It also gives the limitations on extreme positions

4. From Figures 6, 7, and 8, we are able to determine that our mechanism is practically workable if all the parameters are under their limitations.

7. Conclusion

From the mathematical and graphical analysis, we conclude the following

A finite initial velocity is necessary at the equilibrium position to initiate the reciprocating motion. Due to the finite velocity, the bar magnet moves away from it's equilibrium position

As a result, it gets closer to one electromagnet, which now exerts a larger force, and away from the other electro-magnet, which now exerts a smaller force.

When both the forces are repulsive, the bar magnet gets pushed towards the center. When it reaches the center, it still has some kinetic energy which it derives from the work done by the magnetic forces during repulsion. This kinetic energy pushes it on the other side of the equilibrium where a similar process can repeat.

When both the magnetic forces are attractive and the bar magnet is away from its equilibrium position, the attractive force from the closer electromagnet dominates over the one from the other electromagnet. This results in the permanent magnet being pulled in the direction of the initial displacement. As it gets pulled, the magnetic forces do work on it and it gains kinetic energy.

As this process continues, the bar magnet continues to gain kinetic energy, allowing it to move further from the equilibrium position, hence the amplitude of the oscillation grows.

The frequency of the oscillations should preferably be low. We know that it is desirable that the permanent magnet responds to every switch in poles. This cannot happen if the poles switch too rapidly. In that case, the bar magnet may miss some steps or 'slip'.

It is also required that the poles of the electromagnets be out of phase that is one pole be north and the other be south. In other words, both forces be attractive or repulsive. If one force is attractive and the other is repulsive, the bar magnet shall not behave as depicted in figures 6, 7, and 8.

The springs assist further by storing some energy (in the form of potential energy) which gets converted to kinetic energy in further stage.

The only considerable loss in the mechanism is due to friction, which is, more or less unavoidable. The rest of the energy is available for conversion to another form.

This way, the electrical energy supplied to the electromagnets can be converted to kinetic energy of the movable bar magnet.

Hence, an alternative mechanism for conversion of electrical energy to mechanical energy has been developed, which unlike AC motors, produces a reciprocating motion and not rotational motion. It can be compared to the piston motion in an IC engine, where the chemical energy of the fuel being burnt is converted to reciprocation of the piston, which further drives the vehicle.

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